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HEAT AND MASS TRANSFER IN MHD FREE CONVECTION FLOW OVER AN INCLINED STRETCHING PLATE WITH HALL CURRENT

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Abstract: The present paper is an investigation of heat and mass transfer of a steady flow of an incompressible electrically conducting fluid over an inclined stretching plate under the influence of an applied uniform magnetic field. The effects of Hall current are taken into account in this study where the flow is generated due to a linear stretching plate. Using suitable similarity transformations the governing equations of the problem are reduced to couple nonlinear ordinary differential equations and are solved numerically by Runge- Kutta fourth-fifth order method using symbolic software. The numerical results concerned with the effects of various parameters of the flow fields on the velocity, secondary velocity, temperature, and concentration profiles are investigated and presented graphically. The results have possible technological applications in liquid-based systems involving stretchable materials.

Keywords: *MHD, Heat and mass transfer, inclined stretching plate, Chemical reaction, Hall current*

1. INTRODUCTION

The study of boundary layer flow of heat and mass transfer over an inclined stretching plate has generated much interest in recent years in view of its significant applications in industrial manufacturing processes such as glass-fiber, aerodynamic extrusion of plastic sheets, cooling of metallic sheets in a cooling bath, which would be in the form of an electrolyte and polymer sheet extruded continuously from a die are few practical applications of moving surfaces. Glass blowing, continuous casting, paper production, hot rolling, wire drawing, drawing of plastic films, metal and polymer extrusion, metal spinning and spinning of fibers also involve the flow due to stretching surface. Both the kinematics of stretching and the simultaneous heating or cooling during such processes has a decisive influence on the quality of the final products. In recent years, MHD flow problems have become more important industrially. Indeed, MHD laminar boundary layer behavior over a stretching surface is a significant type of flow having considerable practical applications in chemical engineering, electrochemistry and polymer processing. In the extrusion of a polymer sheet from a die, the sheet is sometimes stretched. By drawing such a sheet in a viscous fluid, the rate of cooling can be controlled and the final product of the desired characteristics can be achieved. This problem has also an important bearing on metallurgy where magneto hydrodynamic (MHD) techniques have recently been used. In this regard, Sonth et al. [1] discussed heat and mass transfer in a visco-elastic fluid flow over an accelerating surface with heat source/sink and viscous dissipation and Tan et al. [2] studied heat and mass transfer over an impermeable stretching plate and Sing [3] studied the heat and mass transfer in MHD boundary layer flow past an inclined plate with viscous dissipation in porous medium. The effect of chemical reaction on free-convective flow and mass transfer of a viscous, incompressible and electrically conducting fluid over a stretching sheet was investigated by Afify [4] in the presence of transverse magnetic field. Cortell [5] studied the magneto hydrodynamics flow of a power-law fluid over a stretching sheet.

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Abel and Mahesh [6] presented an analytical and numerical solution for heat transfer in a steady laminar flow of an incompressible viscoelastic fluid over a stretching sheet with power-law surface temperature, including the effects of variable thermal conductivity and non-uniform heat source and radiation. Samad and Mohebujjaman [7] investigated the case along a vertical stretching sheet in presence of magnetic field and heat generation. Jhankal and Kumar [8] studied MHD boundary layer flow past a stretching plate with heat transfer. As many natural phenomena and engineering problems are worth being subjected to MHD analysis, the effect of transverse magnetic field on the laminar flow over a stretching surface was studied by a number of researchers [9]. Hall effects on MHD boundary layer flow over a continuous semi-infinite flat plate moving with a uniform velocity in its own plane in an incompressible viscous and electrically conducting fluid in the presence of a uniform transverse magnetic field were investigated by Watanabe and Pop [10]. Aboeldahab and Elbarbary [11] studied the Hall current effects on MHD free-convection flow past a semi-infinite vertical plate with mass transfer. The effect of Hall current on the steady magneto hydrodynamics flow of an electrically conducting, incompressible Burger's fluid between two parallel electrically insulating infinite planes was studied by Rana et al. [12]. Since the study of heat and mass transfer is important in some cases, in the present paper we studied the Hall effects on the steady MHD free-convective flow and mass transfer over an inclined stretching sheet in the presence of a uniform magnetic field. The boundary layer equations are transformed by a similarity transformation into a system of coupled non-linear ordinary differential equations and which are solved numerically by shooting iteration technique along with Runge- Kutta fourth-fifth order method. Numerical calculations were performed for various values of the magnetic parameter, Hall parameter and the relative effect of chemical diffusion on thermal diffusion parameters. The results are discussed from the physical point of view. Such a study is also applicable to the elongation to the bubbles and in bioengineering where the flexible surfaces of the biological conduits, cells and membranes in living systems are typically lined or surrounded with fluids which are electrically conducting (e.g., blood) and being stretched constantly.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider a two dimensional steady laminar MHD viscous incompressible electrically conducting fluid along an inclined stretching plate with an acute angle γ. X direction is taken along the leading edge of the inclined stretching plate and y is normal to it and extends parallel to x-axis. A magnetic field of strength B0 is introduced to the normal to the direction to the flow. The uniform plate temperature T_w (>T_∞), where T_∞ is the temperature of the fluid far away from the plate. Let *u*, *v* and *w* be the velocity components along the x and y axis and secondary velocity component along the z axis respectively in the boundary layer region.

Under the above assumptions and usual boundary layer approximation, the dimensional governing equations of continuity, momentum, concentration and energy under the influence of externally imposed magnetic field are:

Equation of continuity: $$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equation:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}+g\beta(T-T_\infty)cos\gamma+ g\beta^*(C-C_\infty)cos\gamma-\frac{\sigma B_0^2\mu_e}{\rho(1+m^2)}(u+mw) \tag{2}$$

$$u\frac{\partial W}{\partial x}+v\frac{\partial W}{\partial y}=\nu\frac{\partial^2 W}{\partial y^2}+\frac{\sigma B_0^2\mu_e}{\rho(1+m^2)}(mu-W) \tag{3}$$

Energy Equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{K}{\rho c_p}\frac{\partial^2 T}{\partial y^2}+\frac{\nu}{c_p}\left[\left(\frac{\partial u}{\partial y}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2\right] \tag{4}$$

Concentration Equation:

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-K_0(C-C_\infty)^n \tag{5}$$

Boundary conditions are:

$$u=bx, v=W=0, T=T_w, C=C_w \quad at \;\; y=0$$
$$u=0, w=0, T=T_\infty, C=C_\infty \;\; as \;\; y\to\infty$$

where *u* ,*v* and *w* are the velocity components along x , y and z directions, T , T_w and T_∞ are the fluid temperature, the stretching sheet temperature and the free stream temperature respectively while C , C_w and C_∞ are the corresponding concentrations, *K* is the thermal conductivity, K_0 is the reaction rate constant, m is the Hall parameter, n is the order of reaction, *b* is the stretching rate, C_p specific heat with constant pressure, γ is the angle of inclination , α is the thermal diffusivity, μ is the coefficient of viscosity, ν is the kinematic viscosity , σ is the electrical conductivity, ρ is the fluid density, β is the thermal expansion coefficient, β^* is the concentration expansion coefficient, B_0 is the magnetic field intensity, *g* is the acceleration due to gravity, *D* is the coefficient of mass diffusivity respectively.

We introduce the steam function $\psi(x,y)$ as defined by

$$u=\frac{\partial\psi}{\partial y} \quad and \; v=-\frac{\partial\psi}{\partial x}$$

To convert the governing equations into a set of similarity equations, we introduce the following similarity transformation:

$$u=bxf'(\eta), v=-\sqrt{b\nu}\,f(\eta), W=bxg_0(\eta), \eta=y\sqrt{\frac{b}{\nu}},$$

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \varphi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}$$

From the above transformations, the non-dimensional, nonlinear and coupled ordinary differential equations are obtained as

$$f''' + ff'' - f'^2 + Gr\,\theta\,cos\gamma + Gc\,\varphi\,cos\gamma - \frac{M}{1+m^2}\left(f' + mg_0\right) = 0 \quad (6)$$

$$g_0'' + fg_0' - \left(f' + \frac{M}{1+m^2}\right)g_0 + \frac{Mm}{1+m^2}f' = 0 \quad (7)$$

$$\theta'' + Pr\, f\theta' + Pr\, Ec\left(f''^2 + g_0'^2\right) = 0 \quad (8)$$

$$\varphi'' + Sc\, f\varphi' - Sc\,\zeta\varphi^n = 0 \quad (9)$$

The transform boundary conditions:

$$f = 0, f' = 1, g = 0, \theta = \varphi = 1 \quad at\ \eta = 0$$

$$f = f' = g = \theta = \varphi = 0 \quad as\ \eta \to \infty$$

Where f', g_0, $\theta\, and\, \varphi$ are the dimensionless primary velocity, secondary velocity, temperature and concentration profiles respectively, η is the similarity variable, the prime denotes differentiation with respect to η.

Also the non-dimensional parameters

$$Gr = \frac{g\beta\left(T_w - T_\infty\right)}{b^2 x}, Pr = \frac{\mu}{\alpha}, M = \frac{\sigma B_0^2}{\rho b},$$

$$Ec = \frac{c_p b^2 x^2}{\nu^2\left(T_w - T_\infty\right)}, \zeta = \frac{K_0\left(C_w - C_\infty\right)^{n-1}}{b},$$

$$Gc = \frac{g\beta^*\left(C_w - C_\infty\right)}{b^2 x}, Sc = \frac{\nu}{D}$$

are the Grashof number, Prandtl number, magnetic parameter, Eckert number, reaction parameter, modified Grashof number and Schmidt number respectively.

3. RESULTS AND DISCUSSION

The system of ordinary differential Eqs. (6)–(9) subject to the boundary conditions is solved numerically by shooting iteration technique along with Runge-Kutta fourth-fifth order method using symbolic software. We have formulated the effect of Hall Parameter (*m*), Magnetic parameter (*M*), Prandtl number (*Pr*), Eckert number (*Ec*), Schmidt number (*Sc*), Grashof number (*Gr*) and reaction parameter (ζ) of an incompressible fluid over an inclined stretching plate. The numerical calculation for the distribution of primary velocity, secondary velocity, temperature and concentration across the boundary layer for different values of the parameters are carried out. For the purpose of our computation, we have chosen the various values of parameters. Fig. 1- Fig. 5 depicts the variation of primary velocity profiles for different values of various parameters. From the Fig. 1 it is seen that the velocity starts from maximum value at the surface and then starts decreasing until it reaches to the minimum value at the end of the boundary layer for all the values of magnetic field parameter. It is interesting to note that the effect of magnetic field is more prominent at the point of peak value, because the presence of *M* in an electrically conducting fluid introduces a force called Lorentz force which acts against the flow if the magnetic field is applied in the normal direction, as in the present problem. Thus we conclude that *M* be used to control the flow characteristic. The negligible increasing effect are observed in case of Hall parameter which shown in Fig. 2 but from Fig. 6 it is observed that an increasing Hall parameter leads to noticeable decreasing effect in secondary velocity profile. This is due to the facts that the effective conductivity increases with the increase in Hall parameter which rises the magnetic damping force hence decreasing the velocity. From Fig. 3 it is observed that the primary velocity is increased for Ec but reverse result arises in case of *Gr* which shown in Fig. 5 whereas the primary velocity is unchanged for reaction parameter depicts in Fig. 4. Again Fig. 7 indicates that the secondary velocity increases as increase of inclination of the plate. Fig. 8 illustrates the temperature profiles for various values of *Pr*. It is observed that the temperature decrease as an increasing the values of *Pr* the reason is that smaller values of *Pr* are equivalent to increase in the thermal conductivity of the fluid and therefore heat is able to diffuse away from the heated surface more rapidly for higher values of *Pr*. Hence in the case of smaller *Pr* the thermal boundary layer is thicker and the rate of heat transfer is reduced. Similar result arises for increasing values of inclination parameter are shown in Fig. 9. Again, Fig. 10–Fig. 12 display the concentration profile for various parameters. In Fig. 12 it is observed that concentration profile decrease for increasing values of mass diffusion parameter *Sc* because increasing in *Sc* decreases molecular diffusivity which result decrease of the concentration boundary layer. Hence the concentration of the species is lower for large values of Sc also the concentration profile is decreased for Eckert number and reaction parameter which are depicted in Fig. 10 and Fig. 11.

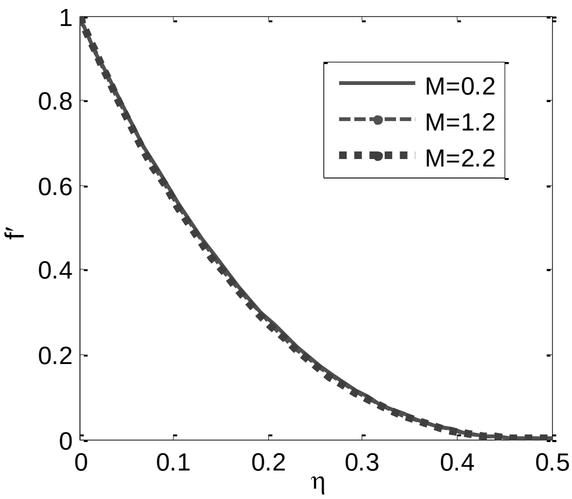

Fig.1: Primary velocity profile for various values of M.

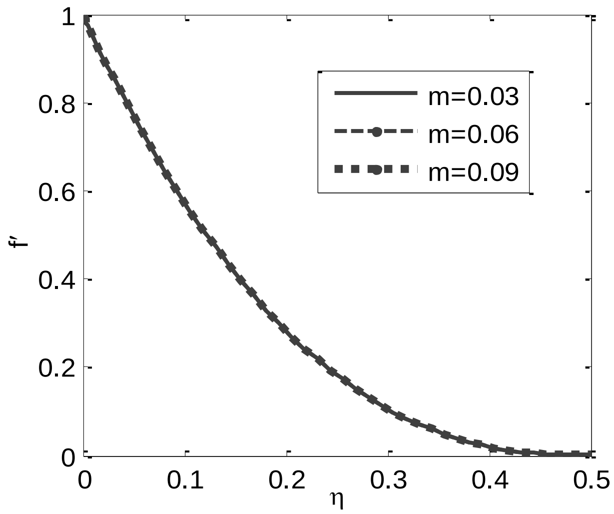

Fig.2: Primary velocity profile for various values of m.

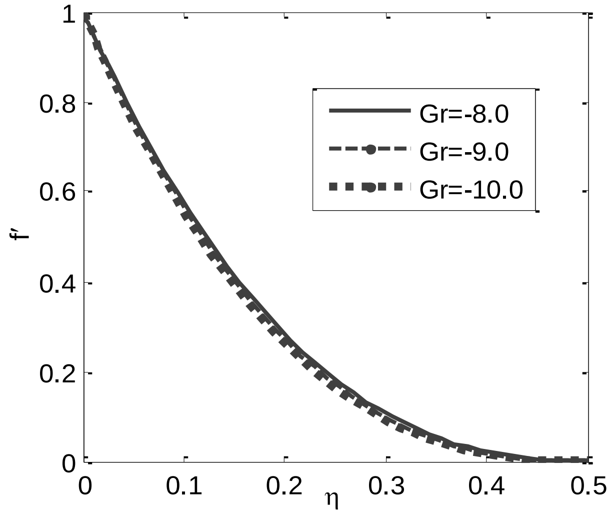

Fig. 5: Primary velocity profile for various values of Gr.

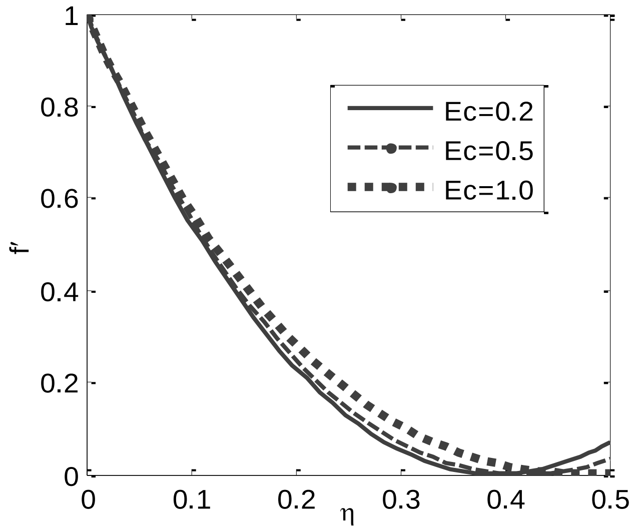

Fig. 3: Primary velocity profile for various values of Ec.

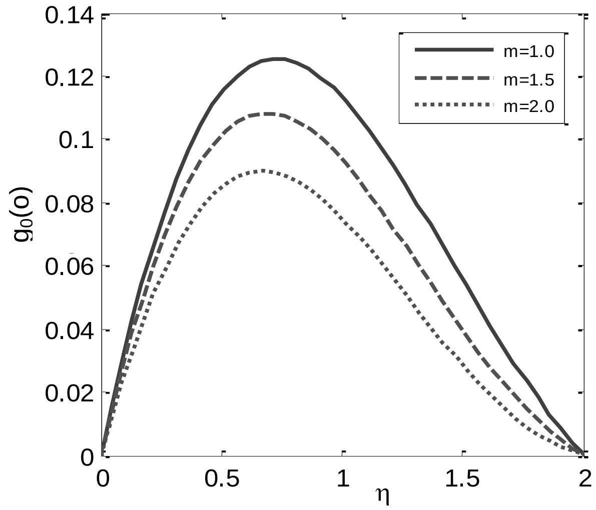

Fig. 6: Secondary Velocity profile for various values of m.

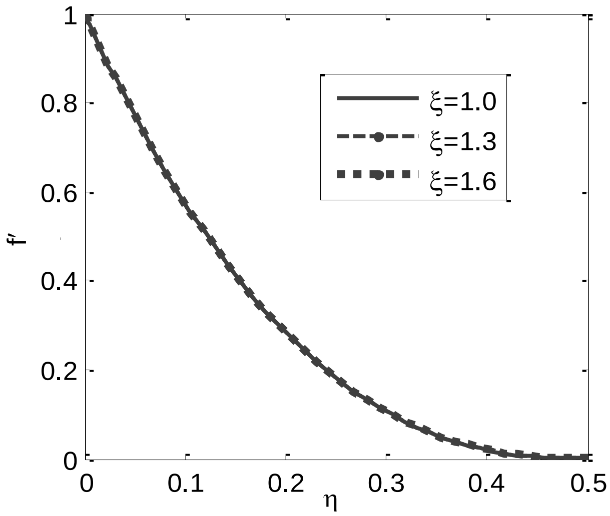

Fig. 4: Primary velocity profile for various values of ζ.

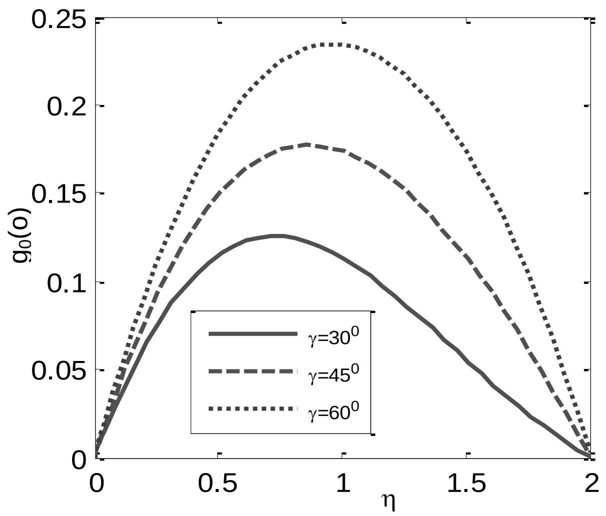

Fig. 7: Secondary Velocity profile for various values of γ.

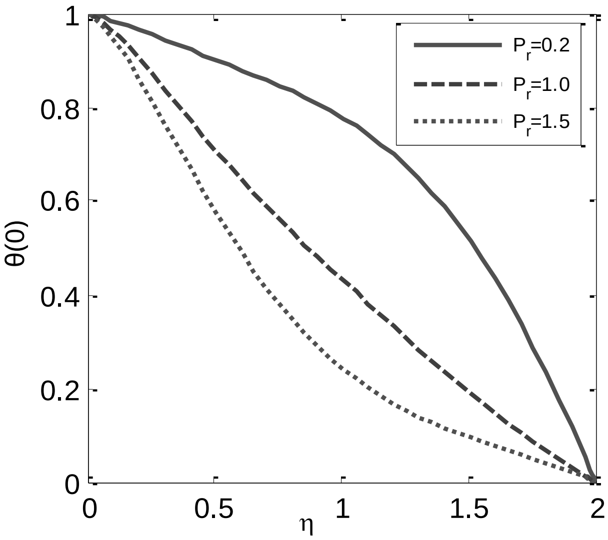

Fig.8: Temperature profile for various values of Pr.

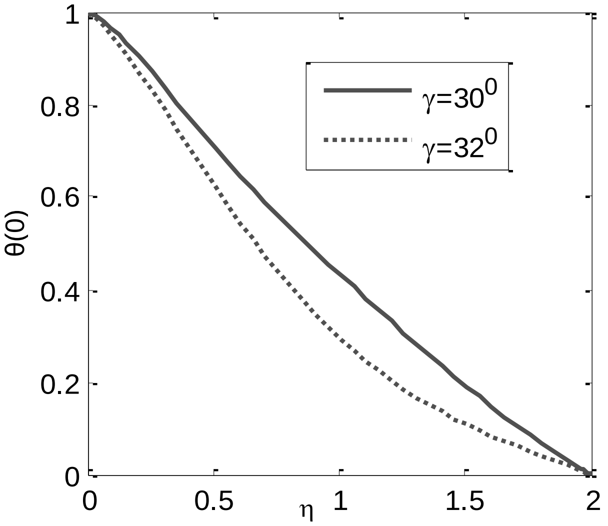

Fig.9: Temperature profile for various values of γ.

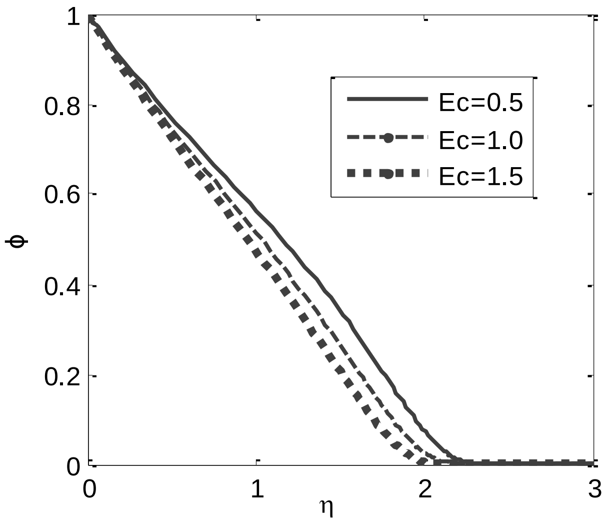

Fig. 10: Concentration profile for various values of Ec.

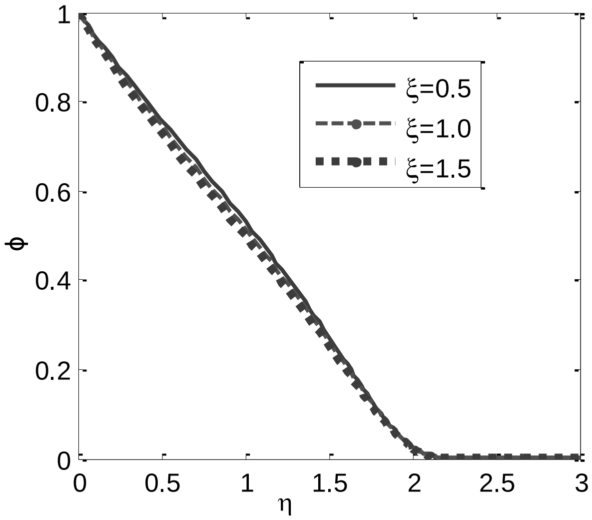

Fig. 11: Concentration profile for various values of ζ.

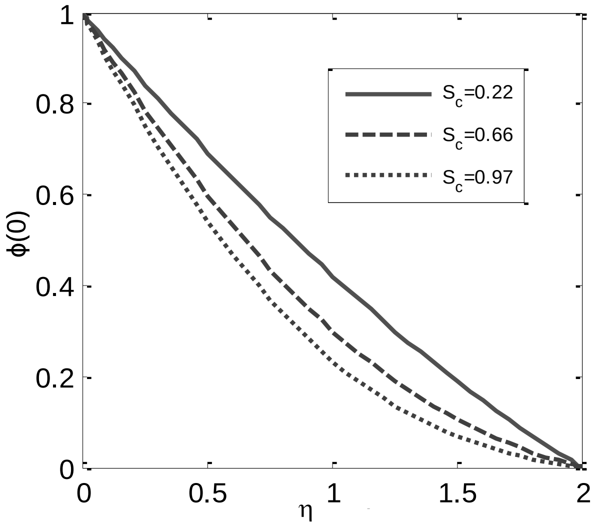

Fig. 12: Concentration profile for various values of Sc.

4. CONCLUSIONS

In this Paper the effects of Hall current parameter on free-convective flow heat and mass transfer of a viscous, incompressible and electrically conducting fluid over an inclined stretching plate have been studied in the presence of magnetic field. Numerical solutions are obtained through shooting method. The observations are that the velocity profile decreases with the increase in the magnetic parameter *M*, which is significant because the presence of *M* in an electrically conducting fluid introduces a force called Lorentz force which acts against the flow if the magnetic field is applied in the normal direction, as in the present problem. Thus we conclude that *M* be used to control the flow characteristic. The negligible increasing effect on primary velocity profile in case of Hall parameter but an increasing Hall parameter leads to noticeable decreasing effect in secondary velocity profiles. This is due to the facts that the effective conductivity increases with the increase in Hall parameter which rises the magnetic damping force hence decreasing the velocity. The temperature profile decrease as an increasing the values of *Pr*, the reason is that smaller values of *Pr* are equivalent to increase in the thermal conductivity of the fluid and therefore heat is able to diffuse away from the heated surface more rapidly for higher values of *Pr*. Hence in the case of smaller *Pr* the thermal boundary layer is thicker and the rate of heat transfer is reduced. Similar result arises for increasing values of inclination parameter. The concentration profile decrease for increasing values of mass diffusion parameter Sc because increasing in *Sc* decreases molecular diffusivity which result decrease of the concentration boundary layer. Hence the concentration of the species is lower for large values of *Sc* also the concentration profile is decreased for Eckert number and reaction parameter.

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